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## Engaging Students in Learning: Mathematical Practices

AN NCTM INTERACTIVE INSTITUTE FOR GRADES 9–12



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# Breakout Workshops

## Building Mathematical Discourse with Beginning Algebra Tasks

(Beginning Algebra)

Struggling algebra students often disengage, making classroom discourse a challenge. Providing small groups of students with related tasks allows for discussion within the group. Comparing and contrasting each related task with other groups' results promotes productive struggle and conceptual understanding of linear functions while breaking the barriers often present in whole-class discussions. Participants will focus on questioning strategies promoting classroom discourse. A laptop or tablet will be helpful during this session.

## Engaging Beginning Algebra Students Using Technology

(Beginning Algebra)

How can I best use the digital devices in my algebra classroom? Participants will explore applications allowing students to investigate mathematical ideas, promoting learning using visual representations of functions, developing mathematical models for algebraic functions, and providing immediate feedback to the student. A laptop or tablet and a TI83/84 calculator will be helpful during this session.

**Speaker:** Jerel Welker

## When Traditional Won't Do: Algebra Tasks that Promote Understanding

(Beginning Algebra)

Not all tasks are engaging tasks. Designing tasks that challenge learners to think deeply and critically about mathematics is a crucial part of the work that teachers do. Learners in Beginning Algebra are often denied access to relevant and rigorous mathematics and the opportunity to engage and reason with mathematics. This session will provide participants with an opportunity to experience tasks designed to engage learners in developing mathematical understanding by connecting to students' lived experiences. Participants will leave this session with strategies for creating and implementing tasks that foster critical thinking, discussion, and collaboration.

## Equity and Data: Beginning Algebra Tasks that Promote Problem Solving

(Beginning Algebra)

Carefully crafted tasks can help teachers engage in mathematics teaching practices that empower learners to recognize and use mathematics as an analytical tool to understand and possibly change society. In this session, participants will engage in tasks designed to examine and critique social, political, and economic issues. In doing these tasks, participants will consider the strategies used by learners to solve the task and how learners can use their mathematical knowledge as a tool for social change. Participants will discuss the mathematics teaching practices from *Principles to Actions* (NCTM 2014), with a particular focus on access and equity.

**Speaker:** Crystal Morton

## Fostering Flexibility with Functions

(Beginning Algebra)

Understanding functions allows students to describe and make sense of relationships between quantities. Students should be able to move flexibly between tabular, graphical, symbolic, and contextual representations of functions. Participants will engage in activities that enhance student understanding of representations of functions and their parameters.

## Opening Up to Algebraic Thinking

(Beginning Algebra)

The study of algebra is viewed by many students as routine equation solving and graphing, consisting primarily of right and wrong answers. However, this view can be changed as teachers pose more "open" questions that allow for different approaches and rich mathematical discourse among students. Participants will explore techniques to adapt existing questions and practical strategies to optimize the effectiveness of discussions.

**Speaker:** Kim Knighton

### **Connecting Algebra and Geometry through Coordinates: Transforming Lines to Make Polygons** (Geometry)

The effects of transformations on lines and the resulting polygons will be analyzed. Participants will look for and make use of mathematical structure to relate this task to future mathematical content. Slopes of lines, translations, reflections, rotations, systems of equations, and relationships in special right triangles will be discussed. Participants will engage in a rich task that makes connections between algebra 1 and geometry. The mathematics teaching practices from *Principles to Actions* will be discussed with an emphasis on facilitating meaningful mathematical discourse.

### **Using Transformations to Make Connections Between Polygons and Circles** (Geometry)

This session will focus on the use of dilations to help students understand similarity. Teachers will explore modeling right triangle similarity by graphing linear equations. A regular hexagon will be constructed (and proved) with the use of compasses or dynamic geometry software, and connections will be made between geometric constructions and polygons graphed in the coordinate plane.

**Speaker:** Benjamin Sinwell

### **Just Keep Spinning: Bridging Geometry to Calculus** (Geometry)

Engage in rich tasks to promote deep understanding of geometric solids. We will use symmetry and transformational geometry to develop volume formulas for cones, cylinders, and frustrums. Experience classroom-tested activities and projects for which students model, create, and construct solids of revolution. These projects and ideas extend to Riemann sums and volumes of solids of revolution in calculus.

### **Construction Junction, What's Your Function?** (Geometry)

Constructions are the foundation for all geometry and provide rich connections between circles, angles, quadrilaterals, and triangles. Experience rich and thought provoking constructions that move student thinking beyond the mundane four basic constructions to deep relationships. These constructions, either with compass and straightedge or with dynamic geometry software, are tactile ways for students to experience, see, and feel the depths and connections within geometry.

**Speaker:** Brian Shay

### **Building a Stronger Foundation for Understanding in Advanced Trigonometry** (Intermediate Algebra)

As topics in algebra 2 become more abstract, establishing and building on strong foundations of understanding can support greater retention and the purposeful application of abstract procedures. Through approaches that build procedural fluency from conceptual understanding, this session seeks to lower the barriers of abstraction that lead students to blindly misapply procedures, not knowing what questions are asking them to do and not knowing whether their answers even make sense. In this session, we will engage with strategies and tasks that both establish a stronger foundation of understanding in right triangle trig and that extend nicely to support a more successful move beyond SOHCAHTOA into nonright triangle trigonometry.

### **Sustaining High Cognitive Demand in Intermediate Algebra Problems** (Intermediate Algebra)

Beyond choosing good problems, teachers should be equipped with strategies for implementing and modifying problems to sustain their cognitive demand for a wide range of diverse learners. This can be particularly difficult in intermediate algebra with problems that lack any plausible "real-life" application or meaningful context. During this session, we will address several questioning strategies for developing additional purposeful questions that create "higher ceilings" or even "different ceilings" to meaningfully extend the runway of any intermediate algebra problem or task.

**Speaker:** Jason Slowbe

### **A Multiple Representation Mash-Up Problem for Intermediate Algebra** (Intermediate Algebra)

We will work on a challenging intermediate algebra task that was created by mashing two classic multiple representation problems. You will leave the session with ideas about how you can transform your favorite activities from early algebra into cognitively demanding tasks that are appropriate for the intermediate algebra classroom.

### **Let's Talk about Intermediate Algebra** (Intermediate Algebra)

This goal of this session is to use a shared problem-solving experience to make sense of larger ideas about facilitating meaningful mathematical discourse in the intermediate algebra classroom. Participants will experience a discourse-intensive intermediate algebra task and debrief the collaborative problem-solving experience in the context of the *Principles to Actions* Mathematical Teaching Practices.

**Speaker:** Nicole Banister

## **Piecewise Functions and the Mathematics Teaching Practices**

(Intermediate Algebra)

Piecewise functions are a natural step in solving many real-world problems. They incorporate understanding of functions, domain, and range. We will work with tasks that build piecewise functions from linear and nonlinear functions, that have multiple entry points, and that use multiple representations. Besides sharing our solutions, we will examine student work for information about their learning. The session will focus on setting goals, selecting tasks, connecting multiple representations, and using discourse and questions to enhance student learning.

**Speaker:** Fred Dillon

## **Building Functions**

(Intermediate Algebra)

Participants will work on two rich tasks that involve creating equations that describe number relationships and building a function that models a relationship between two quantities. They will discuss how the activities promote mathematical discourse.

## **Access and Equity for All Learners**

(Equity)

This session will focus on providing access and equity for ELLs and students with special needs. Participants will engage in a math game to increase mathematical discourse, discuss how to use design activities that allow every student access to high-level mathematics, and use alternative assessments to provide insight into student understanding as well as enhance students writing skills.

**Speaker:** Thomas Duarte

## **Re-Examining Mathematics Teaching Practices with a Lens on Equity**

(Equity)

This workshop will continue to leverage the tasks and activities from the first equitable mathematics instruction workshop. The activities in this workshop will discuss additional case studies and examine problems of practice related to designing and enacting equitable mathematics instruction. We will continue to explore the use of the equity sticky note as a planning tool, and we will also explore teaching strategies like the use of smartness wordless and re-examining discipline practices that inhibit linguistically and culturally diverse students and economically disadvantaged students from fully engaging in the mathematics class.

## **Developing Equitable Teaching Practices**

(Equity)

This workshop features strategies for planning equitable mathematics lessons, which includes posing purposeful questions to encourage sense making and using exit tickets to increase access and participation for all learners. Participants will watch video to identify equitable teaching in action, discuss case studies, and examine problems of practice related to designing and enacting equitable mathematics instruction. Additionally, participants will learn about using a specially designed equity sticky note to plan lessons to facilitate equitable math teaching to increase access to the mathematics content and proficiency with mathematical practices for linguistically and culturally diverse students and economically disadvantaged students.

**Speaker:** Imani Masters

## **Mathematical Practices through a Statistical Lens, Part 1: Statistics versus Mathematics**

(Statistics)

How does rich, productive work in statistics differ from rich, productive work in mathematics? In this first of two 90-minute workshops, we will use a variety of frameworks (Common Core Mathematical Practices, NCTM Process Standards, GAISE Report) to illustrate the similarities and differences between good mathematical work and good statistical work in the 9–12 classroom. We will work through classroom-ready tasks in exploratory data analysis and probability that illustrate how these differences look and sound in a classroom setting. Teachers should bring a laptop or handheld device with Internet access if possible.

## **Mathematical Practices through a Statistical Lens, Part 2: Studies, Simulation, and Inference**

(Statistics)

In this second workshop, attendees will participate in a controlled, randomized experiment. We will do this to better understand how small, thoughtfully designed, nontraditional activities better help students to reason statistically and to apply newly acquired procedural skills appropriately. We will work through an activity that will help participants better understand the essential statistical concepts of random assignment, simulating an experiment based on a randomization model, and making inferences from data to a larger context. Participants do not need to attend part 1 to benefit from part 2.

**Speaker:** William Thill

# Task Discussion Groups

## Algebra 1

### ***Task 1: Analyzing Change***

#### **Promoting Students' Algebraic Thinking Using a Task from Japan**

Participants will explore an open-ended task that provides opportunities for developing linear and quadratic functions from a visual geometric pattern and that will then compare these functions. A major focus of the session will be on attending to the mathematical structures of functions.

### ***Task 2: Counting Trains***

#### **Promoting Students' Algebraic Thinking Using a Hands-On Activity**

Participants will engage in a classroom-tested activity in which linear, quadratic, and exponential functions are developed using patterns built from Cuisenaire® Rods. A major focus of the session will be on using reasoning and sense making to develop the various functions. Participants will also have opportunities to develop and explore extensions to the activity.

### ***Task 3: Analyzing Data—Correlation and Causation***

#### **Promoting Students' Algebraic Thinking Using Data and Statistics**

Participants will use the statistics and probability strand of CCSSM and the Standards for Mathematical Practice by collecting and graphing data that suggests a linear relationship. A major focus of the session will be on finding a line of best fit and assessing how good the linear model fits the data by using the correlation coefficient, the coefficient of determination, and residuals. Participants will also interpret the slope and y-intercept in context and interpolate, extrapolate, and distinguish between correlation and causation.

# Geometry

## ***Task 1: Rotating Squares***

### **Constructing Valid Mathematical Arguments**

Participants will examine the concepts of congruence, transformation, and similarity as they discover a relationship between congruent polygons and their areas. Both abstract and quantitative reasoning will be used as participants make and justify conjectures as well as discuss what it means to provide a valid and formal proof of a mathematical argument.

## ***Task 2: Packaging Soda Cans***

### **Seeing Structure and Making Sense of Problems**

What is the most cost effective, efficient way of packaging soda cans? Participants will apply geometric understandings of area of cross-sections and other circle properties to determine how to best fit circles into rectangles of different sizes. The task employs student use of repeated reasoning and looking for mathematical structure.

## ***Task 3: Constructing Geometric Meaning***

### **Using Tools Effectively**

Participants will examine how integrating other construction tools into their teaching will help students make connections between geometric constructions and broader geometric ideas, including symmetry, congruence, transformation, and reflection. Ideas for appropriate use of tools and structuring effective discussion in the mathematics classroom will be discussed and demonstrated by using a variety of strategies.

# Algebra 2

## ***Task 1: Passing through Points***

### **Building Functions through Given Points**

How many different functions pass through three points? Attendees will collaborate in writing equations of functions that pass through the given points, and they will discover new ways of writing such functions. Attendees will also discuss strategies for implementing this task through the Common Core's Standards for Mathematical Practice in ways that advance students' content knowledge and support them in reasoning and sense making about functions.

## ***Task 2: Adapting to Standards***

### **Building Polynomial Functions**

Creating new resources yourself is difficult, time-consuming, and often unnecessary. Many resources created prior to the Common Core still hold potential as meaningful "Common Core" tasks. Attendees will collaborate in adapting a polynomials task to incorporate the Standards for Mathematical Practice and align the task with the spirit of the Common Core. Interaction between graphical and symbolic representations can promote a better understanding of the mathematics involved. The questioning strategies we will discuss in this session can be leveraged to increase the rigor in any task, empowering teachers to take materials created by others and adapt them for use in their own classrooms.

## ***Task 3: Collecting and Analyzing Data***

### ***Making Connections among Algebra II Topics***

Statistics is a good topic to use to apply previously learned topics in a real-world context, as well as to show connections among different mathematical concepts. The classic M&Ms<sup>®</sup> experiment allows participants to compare different curves of fit made with their own data. The appropriateness of a model will be determined using scatterplots, residual plots, and transformed data. The data collected can be used to make connections between exponential and logarithmic functions and can be used as an impetus to exploring inverse functions. An additional activity about the kidney filtration of blood further enhances understanding of the concepts involved. Sample student work will be provided to allow attendees to reflect on student thinking, to make connections among topics, and to formulate questions that create opportunities for productive classroom discourse.